

## ANALYTICAL DESCRIPTION OF THE SHAPE OF THE PHOTO RESPONSE IN AVALANCHE PHOTODIODES

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### Abstract

*In recent years, a group of Azerbaijani scientists have implemented new detectors with improved micropixel structures, thanks to which they are superior to their counterparts. Micropixel Avalanche Photodiodes (MAPD) which mainly have two structures, deep pixel photodiodes and surface pixel photodiodes. One of the main directions in the development of high-tech photosensors of this type is the modeling and calculation of the parameters of experimental samples. The paper presents the results of an analytical description of the shape of the photoresponse in avalanche photodiodes.*

**Keywords:** MAPD, space charge region, micropixel avalanche photodiodes

### Introduction

Currently, avalanche photodiodes, especially micropixel avalanche photodiodes (MAPD) are widely used in scientific experiments [1–8]. Despite its wide application, an analytical expression describing the characteristics of the photoresponse formation in MAPD has not yet been obtained. This is due to the difficulties of the mathematical solution of the system of equations describing the avalanche process in a semiconductor. In this work, we obtain an analytical expression for the shape of the MAPD photoresponse under certain approximations.

### Analytical work

Let consider a system of equations describing the avalanche process in a sharp silicon p–n+ junction. The system of equations includes the continuity equations for electrons and holes, the Poisson equation, which describes the potential distribution in space charge region (SCR) [9, p.167]. If we take into account the fact that the rate of generation of charge carriers in the SCR of a semiconductor significantly exceeds the rate of their recombination, then the basic equations describing the avalanche process in a semiconductor in a one-dimensional approximation have the form:

$$\begin{aligned}\frac{\partial n}{\partial t} &= \alpha v_s n + \beta v_s n + \frac{1}{q} \frac{\partial J_n}{\partial x} + G \\ \frac{\partial p}{\partial t} &= \alpha v_s n + \beta v_s n - \frac{1}{q} \frac{\partial J_p}{\partial x} + G \\ J_n &= q v_s n \\ J_p &= q v_s p \\ \frac{\partial^2 V}{\partial x^2} &= \frac{q N_a}{\epsilon_s}\end{aligned}\tag{1}$$

In case of following conditions:

$$p(0, t) = 0; n(W, t) = 0; V(x = W) = 0; \frac{\partial V}{\partial x}(x = W) = 0,$$

where  $x$  – counted from the semiconductor surface,  $V$  – voltage applied to the diode,  $q$  – electron charge,  $\epsilon_0 \mu_s$  – semiconductor permittivity,  $N_a$  – concentration of uncompensated ionized acceptor-type impurities,  $G$  – volumetric charge carrier generation rate,  $W$  – maximum thickness of the SCR, achievable upon breakdown of the  $p$ - $n^+$  junction,  $v_s$  – maximum drift velocity of charge carriers,  $n$  – electron concentration,  $J_n$  – current electronic component density,  $\alpha$  – ionization coefficient for electrons. Similar notation takes place for holes.

Under the condition of ionization coefficients for electrons and holes (i.e.  $\alpha=\beta$ ), from the first two equations of system (1) one can obtain:

$$\frac{\partial J}{\partial t} = 2\alpha v_s J + v_s \frac{\partial (J_n - J_p)}{\partial x} + 2q v_s G \quad (2)$$

where  $J = J_n + J_p$  – total conduction current in the SCR region of the semiconductor.

Integrating equation (2) over the thickness of the depleted layer under the initial conditions  $J_n(0)=J_t$ ,  $J_p(0)=0$ ,  $J_n(W)=0$  and  $J_p(W)=J_t$ , we obtain:

$$\frac{J_t}{\partial t} + \frac{2v_s}{W} (1 - \alpha W) J_t = \frac{2v_s i_a}{W} \quad (3)$$

where  $J_t = \frac{1}{W} \int_0^W (J_n + J_p) dx$  – total current,  $i_a = q \int_0^W G(x) dx$  – avalanche-initiating non-stationary current (for example, photocurrent). Here, when integrating equation (3), it was assumed that the ionization coefficient is constant over the entire width of the space charge region of the semiconductor.

Equation (3) has the following two solutions, depending on the value of the expression  $(1 - \alpha W)$ :

$$J_t(t) = \frac{2}{\tau} \exp(-Lt) \int_0^t i_a \exp(Lt') dt' \quad (4)$$

at  $\alpha W \neq 1$ , where  $L = (2/\tau) \times (1 - \alpha W)$ ,  $\tau = W/v_s$  – the time of flight of charge carriers through the semiconductor SCR,

$$J_t(t) = \frac{2}{\tau} \int_0^t i_a dt' \quad (5)$$

at  $\alpha W = 1$ .

Expression (4) shows that when  $\alpha W > 1$ , the initiating current of any shape and duration causes an infinite growth of the avalanche current in the avalanche photodiode.

Of great interest is the process of formation of a pulsed current initiated by a single photoelectron. To describe this process, you can use the expression  $i_a = q \times \delta(t)$ , where  $q$  is the electron charge,  $\delta$  is the Dirac function. In this case, integrating expression (4) and (5), we obtain, respectively

$$J_t(t) = \frac{2q}{\tau} \exp(-Lt) \quad \alpha W < 1 \quad J_t(t) = \frac{2q}{\tau} = \text{const} \quad \alpha W > 1. \quad (6)$$

It can be seen that single photoelectrons can also cause an exponentially decreasing or constant avalanche current, depending on the value of the expression  $\alpha W$ .

Under the condition  $\alpha W < 1$ , the photoresponse of the device decays after the end of the non-stationary avalanche-initiating current. As an example, we can consider the shape of the photoresponse of the device to a rectangular light pulse. The shape of such a rectangular light pulse can be expressed as follows:

$$i_a(t) = I_0 \times \eta(t - t_1) \times \eta(t_2 - t), \quad (7)$$

where  $\eta$  – single function, i.e.  $\eta = 1$  in range  $t_1 < t < t_2$  and  $\eta = 0$  at  $t < t_1$  and  $t > t_2$ .

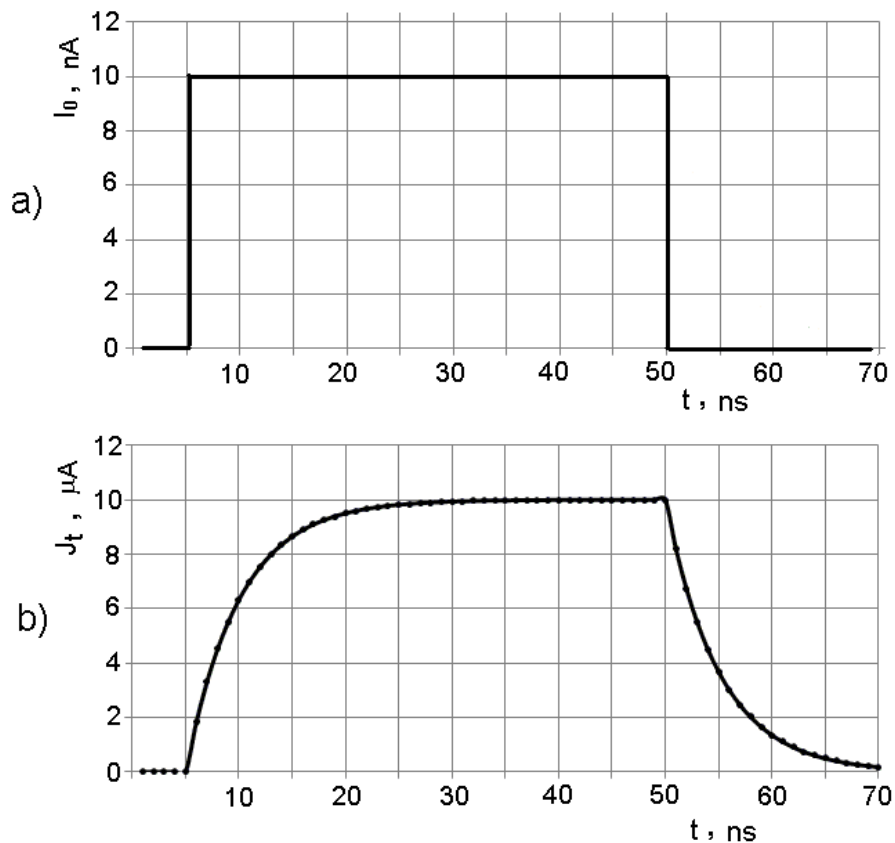
Integrating expressions (4) taking into account (7), we obtain the following expression describing the leading edge of the photoresponse

$$J_t = I_0 M \times (1 - \exp(-\frac{2}{\tau M} \times t)). \quad (8)$$

The trailing edge of the pulse, according to expression (6), is described by the formula

$$J_t = I_0 M \times \exp(-\frac{2}{\tau M} \times t). \quad (9)$$

Fig. 1 shows the photoresponse of an avalanche photodiode during registration of a rectangular light pulse. The following were used in the calculations: amplification factor  $M = 1/(1 - \alpha W) = 1000$ ,  $I_0 = 10 \text{ nA}$ ,  $t_1 = 5 \text{ ns}$ ,  $t_2 = 50 \text{ ns}$ ,  $\tau = 10 \text{ ps}$ .



**Fig. 1.** The shape of the light signal (a) and photoresponse (b) of the avalanche photodiode at a gain of  $M=1000$

It can be seen that the photoresponse fronts are characterized by the integration time of the avalanche process equal to  $\tau_i = (\tau M/2)$ . If we consider the bandwidth of the device  $\Delta f = 1/\tau_i$ , then we get  $\Delta f \times M = \left(\frac{2}{\tau}\right) = \text{const}$ . This means that the bandwidth of an avalanche photodiode operating below the breakdown potential is inversely proportional to the gain of the avalanche process.

#### **Aknowledgment**

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#### **Results**

It should be noted that only the average statistical parameters of the avalanche process were considered above, i.e. for example,  $\alpha W=1$  means that a photoelectron passing through the depletion region of a semiconductor can, on average, create one secondary electron. In this case, a low efficiency of triggering the avalanche process by a single photoelectron is achieved. For example, under the assumption of the Poisson character of fluctuations of the avalanche process, it can be shown that at  $\alpha W=1$  the probability of electrons flying through the SCR without multiplication reaches  $\sim 40\%$ . Therefore, avalanche photodiodes operating under conditions above the breakdown potential, i.e. micropixel avalanche photodiodes (MAPD) have a significant advantage over their counterparts. MAPD are capable of operating at  $\alpha W \gg 1$ , and therefore the avalanche process is much faster.

Thus, an analytical expression for the photoresponse of an avalanche photodiode has been obtained, which makes it possible to study the operating parameters of the device.

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### Selvari fotodiodların fotocavab formasının analitik təsviri

#### Xülasə

*Son illər bir qrup azərbaycanlı alimlər təkmilləşdirilmiş mikro-piksel strukturuna malik yeni detektorları tətbiq ediblər. Mikropikselli selvari fotodiodlar iki əsas struktura malikdir: dərin piksel fotodiodları və səthi mikropikselli fotodiodlar. Bu tip yüksək texnologiyalı fotodiodların hazırlanmasında əsas istiqamətlərdən biri eksperimental nümunələrin parametrlərinin modelləşdirilməsi və hesablanmasıdır. Məqalədə selvari fotodiodların fotocavab formasının analitik təsvirinin nəticələri təqdim olunur.*

**Açar sözlər:** MSFD, fəza yükləri sahəsi, mikropikselli selvari fotodiodlar

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## **Аналитическое описание формы фотоответа в лавинных фотодиодах**

### **Резюме**

*В последние годы группа азербайджанских ученых применили новые детекторы с улучшенными микропиксельными структурами, благодаря которым они превосходят свои аналоги. Микропиксельные лавинные фотодиоды (МЛФД), которые в основном имеют две структуры: глубокие пиксельные фотодиоды и поверхностные пиксельные фотодиоды. Одним из основных направлений в разработке высокотехнологичных фотодатчиков такого типа является моделирование и расчет параметров экспериментальных образцов. В статье представлены результаты аналитического описания формы фотоответа в лавинных фотодиодах.*

***Ключевые слова:** МЛФД, область пространственного заряда, сельварские фотодиоды с микропикселями*

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