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**MATHEMATICAL MODELING OF CIVIL LAW DISPUTES****<sup>1</sup>Rovshan Zulfugar oghlu Humbataliev, <sup>2</sup>Sariya Ingilab gizi Allahverdiyeva**<sup>1</sup>Doctor of Mathematics, Professor  
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**Abstract:** *The paper investigates the possibilities of analyzing and forecasting civil law disputes based on mathematical models. The multifactorial and uncertainty-laden nature of legal decision-making necessitates their representation through formalized models. The study examines the application potential of mathematical approaches such as game theory, probabilistic models, optimization, and fuzzy logic, and analyzes how these methods contribute to the resolution of legal disputes and the assessment of legal risks within a scientific context.*

**Keywords:** *civil law, mathematical modeling, game theory, legal probabilities, optimization, fuzzy logic*

**1. Introduction**

In contemporary legal practice, the analysis of legal disputes is increasingly conducted not only from a jurisprudential standpoint but also through interdisciplinary approaches that enable a more objective and systematic examination. In this context, integrating legal science with other formal and mathematically grounded disciplines-particularly mathematics, statistics, and artificial intelligence-facilitates the rendering of legal decisions in a more precise and predictable manner. This approach expands the boundaries of legal normativity and increases the scope for applying analytical and quantitative methods in legal scholarship.

The process of legal decision-making requires the consideration of numerous variables and factors, analytical evaluation of the interactions between legal subjects, and the selection of optimal outcomes under uncertainty. These characteristics render legal disputes-especially in civil law-structurally complex and multidimensional. The subjectivity of parties' positions, the distribution of the burden of proof, and the probabilistic nature of legal outcomes in civil proceedings necessitate the adoption of novel methodological approaches in legal analysis.

Mathematical modeling allows for the systematization of this complexity and enhances the transparency and objectivity of legal decision-making. Legal processes can be formalized and simulated through decision support systems, multiscale models, game-theoretic frameworks, and probabilistic approaches. AI-based algorithms for automatic analysis of legal precedents and estimation of probabilistic outcomes mark a new phase in the practice of law.

Thus, the application of scientifically grounded mathematical and technological methods in the legal domain contributes not only academically but also practically, enhancing legal efficiency and rationalizing the manifestation of justice. It is worth noting that numerous scholars have contributed to this field. For example:

- In [1], the author describes AI-based systems for the automatic analysis of legal precedents, outlining the formal structuring of legal arguments and text-based legal analytics, as well as decision support and predictive analytics tools for legal professionals.
- In [2], the author models legal disputes as strategic interactions, building game-theoretic models for behaviors such as agreement, concession, and litigation strategy, and explores how game theory can inform legislative design.

• In [3], the author presents optimization tools, Lagrange functions, and nonlinear programming methods for modeling legal decisions, providing a mathematical basis for constructing multifactor models applicable in law.

• In [4], the author demonstrates the use of fuzzy logic in modeling approximate decisions and uncertainty, explaining mechanisms for gradual (as opposed to binary) assessment of legal responsibility and providing practical examples of logical output functions.

• In [5], legal decision-making under uncertainty is modeled through a synthesis of decision theory, behavioral economics, social psychology, and legal theory, enabling the prediction of legal choices.

• In [6], the author showcases early examples of applying fuzzy logic models in legal argumentation, incorporating partial evidence, confidence levels, and graded responsibility into legal analysis, and developing a formal model for decision-making in uncertain cases.

• In [7], the optimization of legal norms is analyzed through efficiency and minimal legal cost principles, combining law and economics in a model-based framework.

• In [8], AI-based expert systems are developed for decision support in civil cases, using automatic analysis of precedents and evidence to prototype legal analytics systems incorporating logical argument structures.

Works [9, 10, 11, 12, 13, 14] develop mathematical models and boundary conditions for predicting corrosion under mechanical stress of ship hull metals in aggressive environments.

Unlike these prior works, this study emphasizes the shift from classical (subjective and narrative-based) approaches to legal decision-making towards mathematical and formalized modeling. Taking into account the multifactorial and uncertain nature of legal decision processes, this research integrates game theory, probabilistic models, optimization, and fuzzy logic into legal analysis. Furthermore, it models legal responsibility not in binary terms (i.e., "guilty" or "not guilty"), but as a gradual construct using fuzzy logic.

Consequently, the concept of partial responsibility is scientifically and mathematically substantiated, contrasting with the traditional binary classification of "full liability" vs. "complete innocence."

Moreover, the behaviors of parties in civil law, imprecision in evidence, and the probabilistic nature of legal consequences are analyzed through formalized models. These models enable the systematization of judicial and legal practitioners' decision-making processes, laying a theoretical foundation for the integration of artificial intelligence and legal technologies. The study proposes a framework for analyzing legal precedents using mathematical models and AI techniques.

## **2. Theoretical Foundations of Mathematical Models in Legal Application**

### **2.1. The Necessity of Formalization**

Legal normative systems often operate through verbal expressions, contextual explanations, and precedent-based decisions. This makes the mathematical and logical systematization of legal texts and judgments challenging. However, the formalization of legal relations—that is, the modeling of legal situations through variables, parameters, and functional relationships—introduces a scientific perspective into the legal decision-making process. Mathematical modeling enables the structuring of legal information, the automation of logical inference, and the identification of cause-effect relationships within legal events.

For example, the conditions for applying a legal norm can be expressed in formal language using variables (e.g., "amount of damage," "degree of violation") and logical operators. In such cases, legal judgments can be derived based on structured input data, thereby increasing the objectivity of legal assessments. Kaplow [13] emphasizes that the mathematical formulation of legal decisions is not only of theoretical interest but also carries high predictive value in practice, especially when these models are calibrated with real legal experience.

The process of formalization lays the groundwork for legal decisions to be made in a more fair, consistent, and transparent manner. Additionally, representing legal reality through

mathematical models allows for the application of experimental, model-based approaches in the study of legal systems.

## 2.2. Modeling Legal Decisions

Modeling legal decisions requires the synthesis of various theoretical and technical approaches. These approaches allow for the formal representation of legal actors' behavior and their decision-making mechanisms. The primary theoretical models used in this domain include:

- **Decision Theory:**

Decision theory explores the principles behind rational choice-making among alternatives by legal subjects. By evaluating the choices of parties involved in legal scenarios through utility functions and risk parameters, this theory facilitates the analysis of legal outcomes. When the rational actor model is applied to legal problem-solving, it plays a foundational role in the economic and utilitarian analysis of legal disputes.

- **Probabilistic Models under Uncertainty (Bayesian Approach):**

Legal realities are often characterized by incomplete information and uncertain conditions. Using the Bayesian approach, prior probabilities can be updated with new evidence to arrive at optimal decisions. This method is particularly applicable in the interpretation of evidence and judicial assessment. For instance, each piece of evidence is treated as information that influences the probability of a legal outcome, and the decision is updated according to Bayes' theorem.

- **Strategic Interaction Models (Game Theory):**

Legal systems frequently involve conflicting interests among parties, and decisions are based on strategic interactions. Game theory serves as a crucial tool for modeling such interactions. This approach mathematically analyzes the possible strategies and interactions of legal actors (judges, attorneys, litigants), illustrating how legal balance may be achieved. For example, settlement negotiations or prosecutorial and defense strategies in criminal cases can be modeled within a game-theoretic framework.

## 3. Game Theory and Civil Legal Disputes

### 3.1. Bilateral Interactions and Strategies

In civil law systems, most legal relationships manifest as bilateral interactions. Particularly in cases such as breach of contract, the behavior of each party is interdependent—each party forms its decision while considering the potential reaction of the other. Such mutual influences create a solid foundation for applying one of the fundamental models of mathematical game theory: the concept of **Nash equilibrium**.

The Nash equilibrium describes a situation in which each party chooses the best possible response given the strategy of the other party, without unilaterally changing their own strategy. In the context of legal disputes, this implies that each party seeks to optimize their actions based on the anticipated behavior of the counterparty.

As an example, in a contract dispute involving two parties—the plaintiff and the defendant—the strategic choices can be defined as follows:

- **For the Plaintiff:**

- Pursue the dispute (initiate legal proceedings)
- Settle (reach an agreement through negotiation)

- **For the Defendant:**

- Accept the settlement offer
- Resist (continue the dispute through litigation)

Based on these strategies, a payoff matrix can be modeled as follows:

	<b>Defendant: Settle</b>	<b>Defendant: Resist</b>
<b>Plaintiff: Settle</b>	(3,3)	(1,4)
<b>Plaintiff: Litigate</b>	(4,1)	(2,2)

Here, each pair  $(x, y)$  represents the multidimensional evaluation of the utilities obtained by the plaintiff and the defendant respectively, depending on the strategy combination. These values may reflect factors such as:

- **Legal Costs:** Expenses related to court proceedings, attorney fees, etc.
- **Reputational Damage:** The impact on public image and standing of both parties.
- **Potential Gains and Losses:** Tangible and intangible outcomes resulting from the legal decision.

The primary advantage of this game-theoretic model is its ability to mathematically capture the interdependence of parties' strategic choices. The Nash equilibrium points—such as (Settle, Settle) = (3,3) or (Litigate, Resist) = (2,2)—can be analyzed and discussed, and the potential movements of the parties toward these equilibria can be studied.

### 3.2. Model Expansion and Application Perspectives

This simple matrix model can be extended into more complex multi-stage games, as well as **probabilistic games** incorporating uncertainty and risk factors. For instance, future actions of the parties as the dispute progresses, time-dependent strategic shifts during legal procedures, and information asymmetry between the parties can all be taken into account.

The game-theoretic approach offers several significant opportunities for legal practice:

- **Effectiveness of Peaceful Settlement Mechanisms:** Game-theoretical models can help identify optimal strategies for dispute resolution, including the design of incentives that encourage parties to settle amicably rather than engage in prolonged litigation.

- **Legislative Regulation and Policy Design:** Modeling the impact mechanisms of legal norms and sanctions can contribute to the development of more effective laws. Game theory provides a framework to assess how changes in legislation affect the strategic behavior of legal actors.

- **Optimization of Judicial Decision-Making:** Theoretical analysis of judges' and attorneys' strategic choices can help make legal proceedings more fair, transparent, and rational.

Furthermore, **calibration of the game's payoff functions** based on real legal data can be performed more accurately using statistical methods and machine learning techniques. This opens up new perspectives in predicting legal decisions and assessing legal risks with greater precision.

## 4. Forecasting Through Statistics and Probability Theory

### 4.1. Evaluating Evidence with the Bayesian Model

In legal systems, the objective and accurate assessment of evidence is fundamental to fair decision-making. In traditional legal practice, however, evidence evaluation often relies on subjective judgment, and factors of uncertainty may not be fully considered. Methods from statistics and probability theory—particularly **Bayesian probability theory**—offer mathematical tools for assessing legal evidence, helping to systematize interpretation and ensure scientifically grounded decisions.

Bayesian theory is based on the core probabilistic principle of updating **prior probabilities** in light of new evidence. This is particularly applicable in legal contexts—for example, refining the likelihood of a contract breach based on initial statistical assumptions and newly presented evidence.

The Bayes formula is given by:

$$P(H/E) = \frac{P(E/H) \cdot P(H)}{P(E)},$$

where  $P(H/E)$  - is the **posterior probability**: the probability of hypothesis (e.g., that a breach of contract occurred) given evidence  $E$ ;  $P(E/H)$  - is the **likelihood**: the probability of observing the evidence  $E$  if the hypothesis  $H$  is true;  $P(H)$  - is the **prior probability**: the initial

probability of  $H$  before seeing evidence;  $P(E)$  - is the **marginal probability** of observing under all possible hypotheses.

Legal disputes often involve uncertain or incomplete information. The Bayesian approach allows for systematic updating of beliefs as new evidence arises, leading to more accurate decision-making. For instance, during litigation, expert reports, witness statements, and other forms of evidence can be evaluated to assess the likelihood of a particular event-such as a contractual breach-occurring.

**Advantages of this approach include:**

- **Information updating:** Probabilities are continuously revised as more evidence becomes available, enabling adaptive decision-making;
- **Modeling uncertainty:** Even when evidence is incomplete or contradictory, probabilistic evaluation allows for objective assessments;
- **Rationalization of legal practice:** The reliability of evidence can be quantified, and the relative weight of different pieces of evidence can be incorporated into the decision-making process.

For example, the prior probability  $P(H)$  could be derived from statistical data on the general frequency of contract breaches. The new evidence  $E$  could be facts indicating that specific contractual terms were violated. Applying Bayes' formula enables courts to compute the updated probability of the hypothesis, thereby strengthening the rationale behind legal judgments.

The Bayesian approach is not limited to evaluating evidence. Broader statistical models are also used for forecasting in legal systems, such as:

- **Regression analysis and statistical tests:** To identify factors influencing the likelihood of legal events (e.g., contract breaches, court rulings);
- **Time series models:** To analyze trends and changes in legal processes over time and to forecast future developments;
- **Machine learning methods:** To analyze large-scale legal databases and automate predictive analytics using statistical and computational algorithms.

These approaches enhance **data-informed, objective, and scientifically grounded legal decisions**, and they play a key role in **legal risk management**.

**5. Legal Resource Management: Optimization**

Effective decision-making in managing legal disputes is a core objective of legal management. The resolution of disputes-through litigation or settlement-can have legal, economic, and reputational implications. Therefore, it is essential to apply mathematical tools such as **linear programming** and **optimal decision theory** to model and optimize legal strategies.

The general **objective function** in this context can be expressed as:

$$\min Z = \alpha C + \beta T + \gamma R,$$

where  $C$  legal costs (e.g., court fees, attorney expenses);  $T$  time resources required to resolve the dispute;  $R$  reputational risks;  $\alpha, \beta, \gamma$  weight coefficients representing the relative importance of each component.

By applying this model, legal managers can gain the following strategic advantages:

- **Comparison of alternative strategies** (e.g., settlement vs. litigation) based on cost, time, and reputational outcomes;
- **Scientific decision-making:** A data-driven alternative to intuitive or subjective choices;
- **Quantification of reputational risks**, allowing them to be formally incorporated into the decision-making process.

This approach can serve as an effective tool for **corporate risk management, strategic legal planning, and financial optimization** within legal departments.

**6. Fuzzy Logic and Uncertain Legal Decisions**

In resolving modern legal disputes, decisions often need to be made under conditions of uncertainty. In such cases, traditional classical logic (which is binary-"true" or "false") may fail to

fully reflect legal realities. **Fuzzy logic** (or soft logic), however, provides the flexibility to model such situations more effectively.

### 6.1. Modeling Cases of Partial Responsibility

Dividing legal liability strictly into “fully responsible” and “not responsible” categories is often inadequate in real cases. Especially in civil law, where contract breaches, breaches of trust, and insufficient evidence are common, defining liability on a **gradual scale** allows for more just decisions.

For example, consider the following uncertain information:

- Probability of the defendant’s contract breach: 0.65
- Intensity of breach of trust: 0.4
- Minimum required evidence threshold for the judge’s decision: 0.7

While these indicators do not fully justify the defendant’s complete liability, they support the conclusion of **partial responsibility**. This situation can be modeled by the following fuzzy function:

$$\mu(x) = \begin{cases} 0, & x \leq 0,5 \\ 2x - 1, & 0,5 < x < 1, \\ 1, & x \geq 1 \end{cases}$$

where a  $\mu(x)$  is the fuzzy function indicating the degree of responsibility, and  $x$  is the normalized value of the total legal evidence and indicators.

This function creates a **transition zone** between full responsibility and full non-responsibility, offering several advantages:

- **Decision-making flexibility** - allows for balanced decisions in uncertain or mixed cases;
- **Evidence-based fair allocation of liability**;
- Provides a **foundation for systematic modeling of legal precedents and practice**.

Fuzzy logic-based approaches are especially effective in alternative dispute resolution, arbitration, and commercial law.

### 7. Conclusion and Prospects

The mathematical modeling of civil legal relations is of significant importance in enriching legal science with formalized approaches. Structuring legal decision-making processes, risk analysis with probabilistic models, and strategy evaluation via game theory open new horizons for modern legal practice. Research in this direction will enhance the analytical skills of legal professionals and contribute to making the legal system more transparent and efficient.

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## MÜLKİ HÜQUQ MÜBAHİSƏLƏRİNİN RİYAZİ MODELƏŞDİRİLMƏSİ

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**Xülasə:** Məqalə mülki hüquq mübahisələrinin riyazi modellər əsasında təhlili və proqnozlaşdırılması imkanlarını araşdırır. Hüquqi qərarvermənin çoxfaktorlu və qeyri-müəyyənliklə yüklənmiş təbiəti bu proseslərin formallaşdırılmış modellərlə ifadəsini zəruri edir. Tədqiqat oyun nəzəriyyəsi, ehtimal modelləri, optimallaşdırma və qeyri-səlis məntiq kimi riyazi yanaşmaların tətbiq potensialını nəzərdən keçirir və bu metodların hüquqi mübahisələrin həllinə, eləcə də hüquqi risklərin elmi kontekstdə qiymətləndirilməsinə necə töhfə verdiyini təhlil edir.

**Açar sözlər:** mülki hüquq, riyazi modelləşdirmə, oyun nəzəriyyəsi, hüquqi ehtimallar, optimallaşdırma, qeyri-səlis məntiq

## МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ГРАЖДАНСКО-ПРАВОВЫХ СПОРОВ

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**Резюме:** В работе исследуются возможности анализа и прогнозирования гражданско-правовых споров на основе математических моделей. Многофакторный характер и присущая неопределённость процесса принятия юридических решений обуславливают необходимость их представления с помощью формализованных моделей. В исследовании рассматривается потенциал применения математических подходов, таких как теория игр, вероятностные модели, оптимизация и нечеткая логика, а также анализируется, каким образом эти методы способствуют разрешению правовых споров и оценке юридических рисков в научном контексте.

**Ключевые слова:** гражданское право, математическое моделирование, теория игр, юридические вероятности, оптимизация, нечеткая логика

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